Solutions to HW1

2.c) $x_n = x_{n-2}$ $\qquad x_n = r^n \Rightarrow r^n = r^{n-2}$

$x_1 = 3 \quad x_2 = 5$ $\qquad r^2 = 1$

$r_\pm = \pm 1$

general solution: $x_n = a \cdot r_+^n + b r_-^n$

$= a + b(-1)^n$

$x_1 = 3: \quad a - b = 3 \qquad \Rightarrow \qquad 2a = 8$

$x_2 = 5: \quad a + b = 5 \qquad\qquad a = 4 \qquad b = 1$

$\therefore \quad x_n = 4 + (-1)^n$ for $n = 1, 2, \ldots$

d) $x_{n+2} = 2x_{n+1}$ $\qquad x_n = r^n \Rightarrow r^{n+2} = 2 \cdot r^{n+1}$

$x_0 = 10$ $\qquad r = 2$

general solution: $x_n = a \cdot 2^n$

$x_0 = 10: \quad 10 = a$

$\therefore \quad x_n = 10 \cdot 2^n$ for $n = 0, 1, 2, \ldots$

6e) $x_{n+1} = -x_n + 3y_n$

$y_{n+1} = x_n / 3$

$y_n = \frac{x_{n-1}}{3} \Rightarrow x_{n+1} = -x_n + \frac{3 \cdot x_{n-1}}{3} = -x_n + x_{n-1}$

$x_n = r^n \Rightarrow r^{n+1} = -r^n + r^{n-1}$

$r^2 = -r + 1$

$r^2 + r - 1 = 0$

$r_\pm = \frac{1}{2}\left(-1 \pm \sqrt{1+4}\right) = \frac{1}{2}\left(-1 \pm \sqrt{5}\right)$

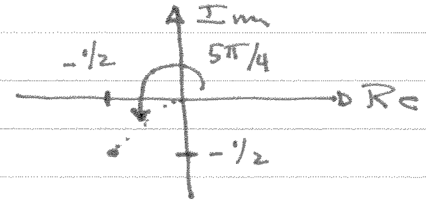
general solution:

$$x_n = a \cdot r_+^n + b r_-^n = a \cdot \left[\tfrac{1}{2}(-1+\sqrt{5})\right]^n + b\left[\tfrac{1}{2}(-1-\sqrt{5})\right]^n$$

$$y_n = \tfrac{1}{3} x_{n-1} = \tfrac{1}{3}\left\{ a \cdot \left[\tfrac{1}{2}(-1+\sqrt{5})\right]^{n-1} + b\left[\tfrac{1}{2}(-1-\sqrt{5})\right]^{n-1} \right\}$$

HW1
#2

8c) $z = -\frac{1}{2} - \frac{1}{2}i$



$r = \sqrt{x^2+y^2} = \sqrt{\frac{1}{4}+\frac{1}{4}} = \frac{1}{\sqrt{2}}$

$\theta = \frac{5\pi}{4}$

$$z^n = r^n e^{in\theta} = \left(\frac{1}{\sqrt{2}}\right)^n e^{in\cdot\frac{5\pi}{4}}$$

$$= \left(\frac{1}{\sqrt{2}}\right)^n \left[\cos\left(n\cdot\frac{5\pi}{4}\right) + i\sin\left(n\cdot\frac{5\pi}{4}\right)\right]$$

9b) $x_{n+2} - x_{n+1} + x_n = 0$

$x_n = r^n \Rightarrow r^2 - r + 1 = 0 \Rightarrow r_\pm = \frac{1}{2}\left(1 \pm \sqrt{1-4}\right)$

$= \frac{1}{2}\left(1 \pm i\sqrt{3}\right)$

general solution: $x_n = a\cdot r_+^n + b r_-^n$

$= a\left[\frac{1}{2}(1+i\sqrt{3})\right]^n + b\left[\frac{1}{2}(1-i\sqrt{3})\right]^n$

16 a) $R_{n+1} = \underbrace{(1-f)R_n}_{\text{\# that survive to next day}} + \underbrace{M_n}_{\text{\# added by bone marrow}}$

$M_{n+1} = \underbrace{\gamma \underbrace{f\cdot R_n}_{\text{\# that are lost in a day}}}_{\text{production of new cells}}$

b) Also, $M_n = \gamma f R_{n-1} \Rightarrow$

$R_{n+1} = (1-f)R_n + \gamma f R_{n-1}$

b) $R_n = r^n \Rightarrow r^2 = (1-f)r + \gamma f$

$r^2 - (1-f)r - \gamma f = 0$

$r_\pm = \frac{1}{2}\left[1-f \pm \sqrt{(1-f)^2 + 4\gamma f}\right]$

HW1 #3

$$r_+ = \frac{1}{2}\Big[1-f+\underbrace{\sqrt{(1-f)^2+4\gamma f}}_{\gamma f>0 \Rightarrow \sqrt{\ } > 1-f}\Big]$$ non-negative mag $= r_+$

$$r_- = \frac{1}{2}\Big[1-f-\underbrace{\sqrt{(1-f)^2+4\gamma f}}_{\geq 1-f}\Big]$$ non-positive mag $= -r_-$

c) The general solution is

$$R_n = a\cdot r_+^n + b r_-^n$$

Note $0 \leq |r_-| \leq |r_+| = r_+$, so for $R_n \to$ constant it's necessary that $r_+ = 1$

$r_+ = 1$:
$$1-f+\sqrt{(1-f)^2+4\gamma f} = 2$$
$$\sqrt{(1-f)^2+4\gamma f} = 1+f$$
$$1-2f+f^2+4\gamma f = 1+2f+f^2$$
$$4\gamma f = 4f$$
$$\Rightarrow \quad f=0 \ \underline{\text{or}} \ \gamma = 1$$

Assuming $f=0$ not possible then $\gamma = 1$

d) $\gamma = 1 \Rightarrow$
$$r_- = \frac{1}{2}\Big[1-f-\sqrt{(1-f)^2+4f}\Big]$$
$$= \frac{1}{2}\Big(1-f-\sqrt{1+2f+f^2}\Big)$$
$$= \frac{1}{2}\Big[1-f-\sqrt{(1+f)^2}\Big]$$
$$= \frac{1}{2}\Big[1-f-(1+f)\Big] = -f$$

general solution:
$$R_n = a + \underbrace{b\cdot(-f)^n}_{\text{assuming } 0<f<1 \text{ then this alternates in sign \& goes to zero}}$$

as $n\to\infty$ this will be decayed oscillation, around $R_n = a$, as $n \to \infty$